9 Design of Multivariable Industrial Control Systems by the Method of Inequalities

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Abstract. This chapter reviews the application of the method of inequalities to the multivariable control of important industrial processes such as distillation columns, a turbofan engine and power systems. It is shown that desired performance and stipulated constraints can be expressed in terms of a conjunction of inequalities, any solution of which gives an acceptable design. The method of inequalities is consistently shown to give an efficient solution to the given problem, not only in terms of the ease of problem formulation, but also because of the simple and implementable controllers obtained.

9.1 Introduction

The design of a range of challenging industrial control processes, carried out with the method of inequalities, is reviewed in this chapter. All the designs are in accordance with the conventional definition of control and are subject to standard design criteria expressed in the form of inequalities.

The first process considered is the distillation column, which is used widely (Rademaker *et al* 1975, Shinskey 1977, Skogestad and Postlewaite 1996) in the process industries to separate mixtures with different volatilities. The economic incentives for investigating the feasibility of controlling, simultaneously, more than one product composition of a distillation column become obvious when it is realised that both capital and energy costs are considerably reduced if more than one product meeting specifications, can be obtained from a single column since there is then no need for further processing.

When considering multivariable control of distillation columns, the situation of dual product control is by far the most common in the literature, hence this situation will be considered first. The possible use of decentralised control involving the use of diagonal proportional-plus-integral (PI) controllers to control both product compositions has long been a controversial topic (Rosenbrock 1962, Niederlinski 1971a). Rosenbrock (1962) used somewhat arbitrary transfer functions to demonstrate that de-centralised control will often result in severe interaction. Ridjnsdorp (1965) and Davison (1970) pursued the problem further by outlining the conditions that give rise to interaction. While Ridjnsdorp (1962) and Rademaker *et al* (1975) recommended

ratio control, Luyben (1970) proposed non-interacting control (Waller 1974, Taiwo 1980) as a means of eliminating interaction.

Wood and Berry (1973) then compared the merit of both ratio control and non-interacting control experimentally. They concluded that although non-interacting control is to be preferred to ratio control, either scheme is superior to the scheme, which uses two conventional controllers and ignores the interaction present in the plant. Luyben and Vinante (1972) also carried out some experimental investigation of non-interacting control, while others (Waller and Fagervik 1972, Nakanishi *et al* 1974, Schwanke *et al* 1977) present only simulation results.

One drawback of non-interacting control is that system performance is very sensitive to the accuracy of the model (Waller and Fagervik 1972) and as noted by Niederlinski (1971a, 1971b) and Foss (1973) complete decoupling is not necessary in a regulator. It has been used in distillation only as an artifice to enable controllers to be designed using well-known single-variable methods. On the other hand, ratio control, although it can be designed without first determining the plant transfer function, ignores the effects of control action at the top of the column on the composition of the bottom. Also, the accurate measurement of the top vapour flow rate might be difficult, as it usually contains some liquid. In any case, a computer will generally be required to implement either scheme.

This has led process control engineers to investigate how the parameters of two conventional controllers can be determined for decentralised control. Most of the methods used to date are heuristic. Typically the gains of the PI controller, which have been independently tuned for the diagonal elements of the plant transfer function matrix, are successively reduced until the entire system is asymptotically stable. However, this approach often fails to give a system with satisfactory transient performace, see for example Wood and Berry (1973), Luyben and Vinante (1972), Waller and Fagervik (1972), Schwanke *et al* (1977). Another heuristic method, which is slightly more sophisticated and is proposed by Niederlinski (1971a, 1971b), determines the pairing of manipulated and controlled variables such that the gain-bandwidth product for the entire system is maximised. Since this method uses only controllers with integral terms, system steady state performance is generally satisfactory. However, the method has no means of determining controller parameters that ensure that transient interaction is small. In fact, as noted by Niederlinski (1971a) the method may be best suited for determining an initial set of controller parameters which might have to be improved upon using other methods. The progress that has been made in the on-line tuning of decentralised PID controllers has come from three lines of thought. The first method uses relay feedback method (Åström and Hagglund 1984, 1995) in conjunction with sequential loop closure and tuning (Yu 1999, Tan *et al* 1999, Loh *et al* 1993, 1994). An extension of this method(Halevi *et al* 1997, Palmor *et al* 1995) entails connecting relay to all the feedback loops

simultaneously and using ultimate frequency data in PID controller tuning according to Ziegler-Nichols or modified Ziegler-Nichols rules. The second method, called the BLT (biggest log-modulus tuning) method (Monica *et al* 1988, Luyben and Luyben 1997) detunes the Ziegler-Nichols parameters by certain recommended factors until both stability and intuitive cum empirical performance criteria are satisfied, while for the third category, several design methods have been developed for decentralised PI control systems based principally on Nyquist stability criterion and frequency response information to obtain satisfactory closed loop performance. Hovd and Skogestad (1994) and Skogestad and Postlethwaite (1996) employed sequential loop closure and tuning such that the performance criterion is minimised at each design step. Ho *et al* (1997) developed a design method for decentralised PID control systems based on shaping the Gershgorin bands, so that the gain and phase margin specifications for the Gershgorin bands are satisfied. Lee *et al* (1998) extended the iterative continuous cycling method for SISO problems to decentralised PI controller tuning. Chen and Seborg (2003) proposed a method of decentralised PI control based on the idea of independent design, reduction of loop interactions and ensuring system stability. In order to use this method, the plant must be (made) diagonally dominant. The results of applying these methods generally lead to improved controller parameters, although typically, the closed loop systems still suffer from considerable interaction (Balachandran and Chidambaram 1997). It is therefore concluded that reliable design techniques, such as the Ziegler-Nichols method for single-variable systems, have yet to be used in this context. It is also clear that the multivariable model of the plant has to be used for controller design whenever interaction is significant.

Accordingly, the main aim of all the work reviewed here is to apply the computer-aided design method developed by Zakian and known as the method of inequalities (Zakian and Al-Naib 1973, Zakian 1979) to various industrial systems. The effectiveness of the method has been demonstrated in various applications (Taiwo 1978a, 1978b, 1979a, 1979b, 1980,1991, Coelho 1979, Gray and Al-Janabi 1975, 1976, 1991, Bollinger *et al* 1979). In the present application, both the diagonal PI controller and the full PI controller are designed for a transfer function model of a packed distillation column. The results of implementing the diagonal PI controller are then presented. In order to establish the generality of the results obtained for the packed distillation column, the method of inequalities is also applied to three other distillation columns taken from the literature (Wood and Berry 1973, Schwanke *et al* 1977, Luyben and Vinante 1972). Designs obtained using the method of inequalities are compared with the non-interacting design. This comparison is made because process engineers commonly employ non-interacting control (Ridjnsdorp and Seborg 1976).

In previous work, investigators (Schwanke *et al* 1977, Munro and Ibrahim 1973, Pike and Thomas 1974, Davidson 1967, Hu and Ramirez 1972, McGin-

nis and Wood 1974) have used frequency response methods (Rosenbrock 1974, MacFarlane and Belletrutti 1973), optimal control techniques and modal analysis to design multivariable controllers for the plant. However, these investigations were primarily aimed at obtaining multivariable controllers represented by a full matrix (although Rosenbrock's method can be used more flexibly (Maciejowski, 1989)). Also, the method of inequalities has already been shown (Taiwo 1978, 1979, 1980) to have advantages over these other methods in applications.

Other workers, who have applied the method of inequalities to the control of distillation columns, include Chidambaram and co-workers (Srinivasa and Chidambaram 1991, Balachandran and Chidambaram 1996, 1997). In these cases, both single-variable and multivariable models were considered. For the multivariable situations (Balachandran and Chidambaram 1996, 1997) the emphasis has been on the use of decentralised controllers for two-input two-output and higher dimensional systems. They came up with the conclusion that the method of inequalities outperforms other methods such as BLT method (Luyben and Luyben 1997) and the improved sequential loop tuning method of Hovd and Skogestad (see, for example, Hovd and Skogestad 1994, Skogestad and Postlethwaite 1996), both in terms of the simplicity of problem formulation and system performance.

Another very successful area of application of the method of inequalities is in the control of a 24^{th} order, three-input three-output advanced turbofan engine model (Sain *et al* 1978, Taiwo 1979). All aspects of desired performance including the rate of change of controller outputs were successfully incorporated into the problem formulation and very simple controllers were obtained meeting all the desired performance and stipulated constraints. Crossley and Dahshan (1982) applied the method of inequalities to design a longitudinal ride-control system for a STOL aircraft. The controller ensures the comfort of passengers and crew by appropriate reduction of normal acceleration. Furthermore Kreisselmeier and Steinhauser (1983), see also, Maciejowski (1989), in a very challenging application, formulated the problem of designing a robust controller capable of ensuring desired performance in all the flight regimes of a fighter aircraft as a conjunction of 42 performance objectives, which were successfully solved. Their control system design formulation is in accordance with the principle and the method of inequalities (see Chapter 1) but they use a different method for solving the inequalities.

The method of inequalities has also been used to design simple multivariable controllers for realistic models of power systems. Taiwo (1978) used the method of inequalities to design simple controllers for the regulation of terminal voltage and load angle of a turbo-alternator. The results were found to be favourable in comparison to those obtained by Ahson and Nicholson (1976), who used the Inverse Nyquist Array method, in terms of the simplicity of the problem formulation, system performance and simplicity of the controllers obtained using the method of inequalities.

Taiwo (1979b) applied the method of inequalities to the multivariable control of an unstable model of the continuous stirred tank reactor whose operating point has been determined using steady state optimisation. By deriving the formulae for the controller outputs, he showed how the method of inequalities can be used to design controllers such that the responses of both the linear and nonlinear models coincide.

Recently, Zhang and Coonick (2000) have used the method of inequalities for the coordinated synthesis of Power System Stabiliser parameters in multi machine power systems in order to enhance overall system small signal stability. They used genetic algorithms to search for the appropriate controller parameters. As usual, the potency of the method in facilitating precise problem formulation as well as the simplicity of controllers was demonstrated. In particular, it was found that the method of inequalities was very effective in designing decentralised controllers for the plant.

9.2 Application of the Method of Inequalities to Distillation Columns

The method of inequalities is used to design controllers for dual quality control of four binary distillation columns whose top and bottom compositions are controlled by manipulating reflux rate and steam. Two types of multivariable proportional plus integral controllers are designed and the performance of the system with these controllers is compared with that of non-interacting control. The results of implementing the simpler controller, which is represented by a diagonal matrix, on a computer-controlled packed distillation column are also presented. In the sequel, we first describe the experimental setup of the packed distillation column.

9.2.1 The Plant and Control System

The packed distillation column used for the experiments is part of the pilot plant in the department of chemical engineering at UMIST (Stainthorp 1970). This continuous fractionating column separates a mixture of methanol and iso-propanol into 97wt.% pure products.

The column is constructed from mild steel 23cm in diameter and packed with 13mm ceramic Raschig rings to a total height of 8.52m. The products taken off at the top and bottom of the column are returned to the appropriate feed tank for recycling (see Figure 9.1). The reboiler is heated by steam at 4.5bar and the total condenser is cooled by water. Both the reflux and the steam flow rates are controlled by electro-pneumatic valve positioners and monitored by orifice plates fitted with differential pressure cells. Temperatures are measured using platinum resistance thermometers which in the case of the top of the column is connected to a continuous transmitter (accuracy $\pm 0.1°$C) and at the bottom is connected to a Kent six-point recorder, which scans each temperature at 30 sec intervals (accuracy$\pm 0.2°$C).

The plant can be controlled either from a conventional analogue instrument panel or by digital computer (the latter has been used in the experiments reported here), which has a control panel, two teleprinters, a paper tape punch and a paper tape reader.

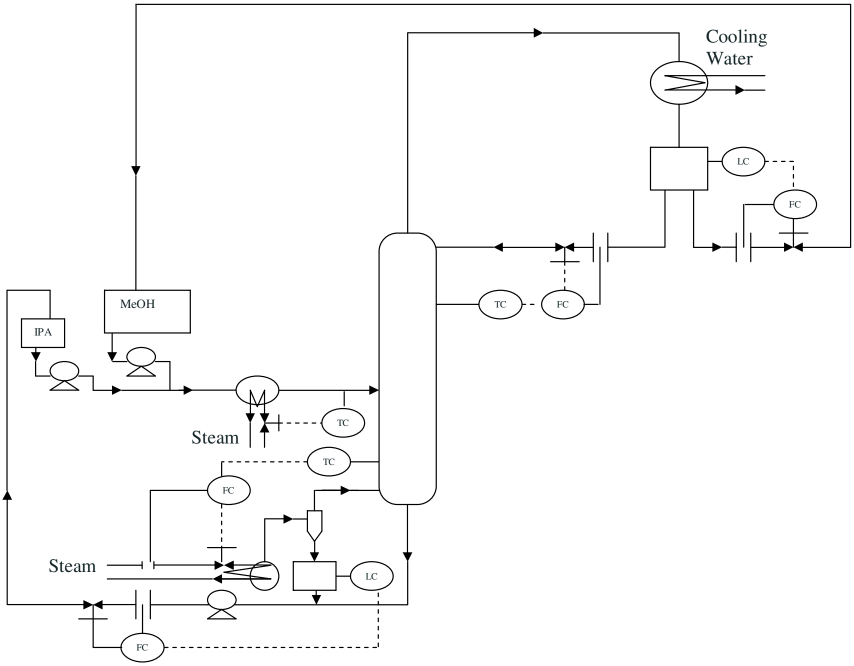

Fig. 9.1. Schematic diagram of the distillation column

The computer is an Argus 308 with 8k core store (24 bit word) and an adequate number of digital to analogue and analogue to digital (10 bits) converters for interfacing to the column and two other similar plants. Organisation of the computer task is achieved by the time-sharing executive program, which also provides facilities for timing and logging operations.

9.2.2 Transfer Function Description of the Packed Distillation Column

The pertinent steady state parameters, which characterise the normal operating point of the plant are given in Table 9.1

It is assumed here that this operating point is based on the economics of the process. The purpose of the control system is then to ensure that

Table 9.1. Steady state operating conditions

	Feed	Distillate	Bottoms	Reflux	Steam
Flow rate (Kg/hr)	61	16	45	48	45
Composition (wt % methanol)	28	97	2	97	-
$T_1 = 68°\mathrm{C}$, $T_2 = 77°\mathrm{C}$, Feed temperature = 69°C					

excursions of system variables from these steady state values are sufficiently small, in spite of the disturbances acting on the system.

Since this is a binary distillation column, composition at a point is uniquely determined by the temperature and pressure at that point. So, by keeping the pressure constant, composition can be inferred from temperature.

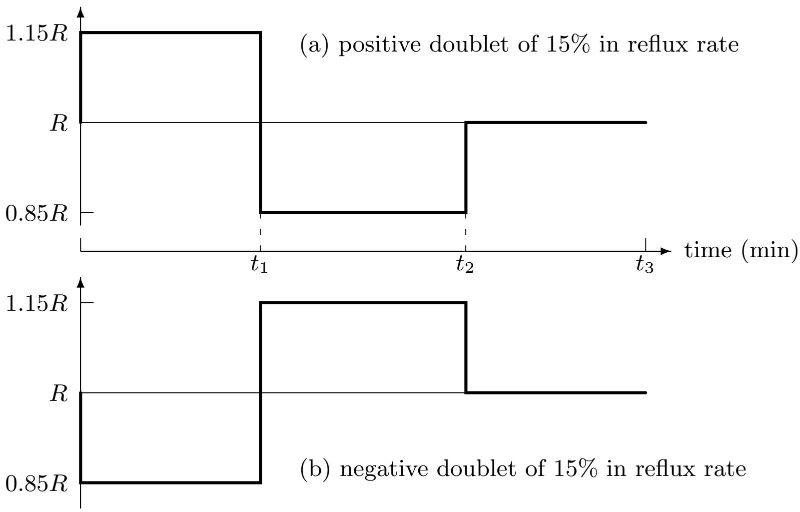

Fig. 9.2. Diagram of positive and negative doublets

In each test the top temperature (T1) and the pressure-corrected bottom temperature (T2) of the plant resulting from "doublet" type (see Figure 9.2) disturbances of reflux and steam rates were recorded every minute. By using this type of disturbance, plant variables are assured to be close to their steady state values, thereby justifying the linearised description; the dangers of driving materials to one end of the column is avoided and plant responses to disturbances in both directions of increasing and decreasing temperatures are combined to elucidate its transfer function in each test.

Preliminary experiments indicated that plant responses were very much obscured by noise when doublet variations were less than ±10%. Conse-

quently, doublet sizes of between ±15% and 20% were adopted. Typical values for t1, t2 and t3 (Figure 9.2) were respectively 15, 30 and 45.

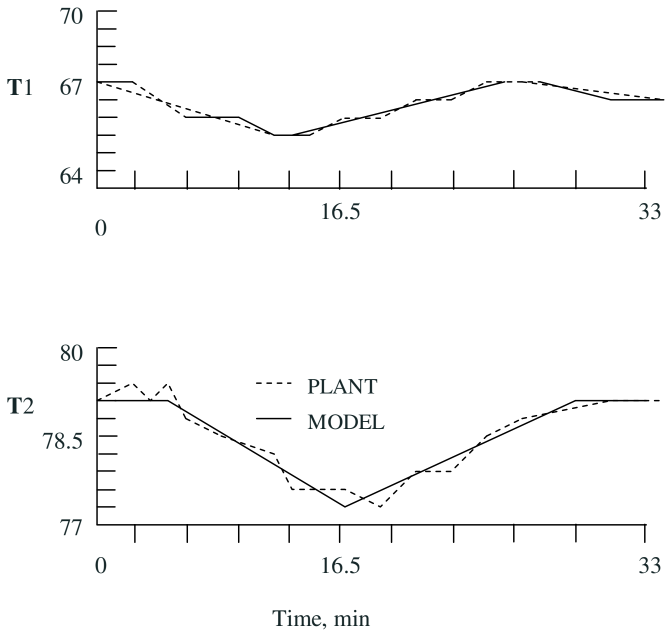

Fig. 9.3. Responses of upper and lower temperatures to a positive doublet of 15% reflux

A least-squares fit employing Rosenbrock's (1960) direct search technique was used to determine the parameters of simple models which describe plant responses. The model adopted for the column is given by

$$\begin{bmatrix} T_1(s) \\ T_2(s) \end{bmatrix} = \begin{bmatrix} g_{11}(s) \; g_{12}(s) \\ g_{21}(s) \; g_{22}(s) \end{bmatrix} \begin{bmatrix} R(s) \\ S(s) \end{bmatrix} \tag{9.1}$$

where

$$\begin{aligned} g_{11}(s) &= \frac{-0.86e^{-s}}{35.4s+1}, \quad g_{12}(s) = \frac{0.6}{(18.9s+1)(2.6s+1)} \\ g_{21}(s) &= \frac{-1.5e^{-4s}}{74s+1}, \quad g_{22}(s) = \frac{1.22}{30.6s+1} \end{aligned} \tag{9.2}$$

The time constants and time delays are in minutes and the gains have the units Khr/kg. Figure 9.3 shows typical agreement between plant outputs and assumed model.

The main difficulties encountered during the identification were due to variations in steam quality and the frequent malfunctioning and small accuracy of the Kent six-point recorder. The steam plant which conveyed steam to the plant also supply adjacent offices and factories, and steam pressure varied with the number of other users. For example, the steam quality was particularly low during lunch time. The best results from the pulse tests were generally obtained at night. Also, because of chanelling caused by interfacial tension between the liquid and the packing surface, the column was not always in thermal equilibrium. The temperature recorded by the thermocouples would therefore be slightly different depending on whether they are in contact with vapour or liquid.

9.2.3 Performance Specifications and Constraints

The purpose of this section is to explain why it is valid to specify performance in terms of the functionals of the system step responses. The explanation is provided by the work of Zakian (1978, 1979), which systematises the ideas relating to performance and sensitivity.

In order to behave properly as a regulator or servomechanism, every component e_i(t) of the error vector in the linear, time-invariant ℓ-input, ℓ-output control system shown in Figure 9.4 should remain sufficiently close to zero throughout the time domain in spite of any disturbances acting on the system or any changes in the characteristics of the plant.

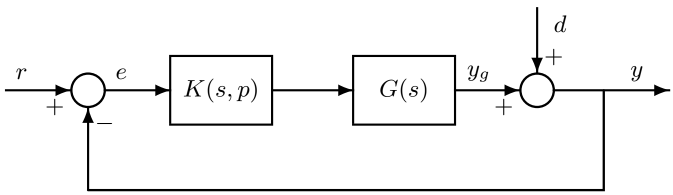

Fig. 9.4. Block Diagram of the feedback structure for the column

Let $J = \{1, 2, \ldots, l\}$, then with regard to Figure 9.4, it can be shown that for every $i \in J$ and $t \geq 0$

$$e_i(t) = \sum_{j=1}^{i} [\delta_{ij} - \sigma_{ij}(t)] \, f_j(0) + \sum_{j=1}^{l} \int_0^t [\delta_{ij} - \sigma_{ij}(\lambda)] \, f_j^{(1)}(t - \lambda) d\lambda \quad (9.3)$$

where $f = r - d$, σ_{ij} is the ith system output due to a unit step change in r_j, while δ_{ij} is the Kronecker delta, defined by

$$\delta_{ii} = 1 \text{ and } \delta_{ij} = 0, i \neq j \quad (9.4)$$

Other symbols are defined in the notation.

Clearly, $e_i(t) = 0$ if a controller can be designed such that for all $t \geq 0$

$$\sigma_{ii}(t) = 1 \text{ and } \sigma_{ij}(t) = 0, i \neq j \tag{9.5}$$

This is not possible in any real system because

$$e_i(0) = f_i(0) \tag{9.6}$$

which shows that the initial value of the error is independent of the controller parameters.

A further reason why (9.5) may not be satisfied is that control signals must be bounded. However, a system with response σ_{ii} which is sufficiently step-like and σ_{ij} which is small for $i \neq j$ is a satisfactory system. It is noted also from (9.3) that $\lim\limits_{t\to\infty} e_i(t)$ exists for all f which tends to a constant as $t \to \infty$ if, $(\sigma_{ij}(\infty)) = I$, that is to say there is no steady-state error in the system step responses. In practice, a small initial period t_s (which is to be distinguished from the settling time) is allowed for the process to "recover" from initial transients and the design aim is then to determine the controller $K(s, p)$ such that the absolute value of the error is less than a specified quantity after t_s, *i.e.*,

$$|e_i(t)| \leq k_i > 0, \quad t \geq t_s \tag{9.7}$$

Experience shows that the settling times of σ_{ii} and σ_{ij}, $i \neq j$ are usually similar even when only the settling time ϕ_{i1} of σ_{ii} is specified in the design. It is therefore sufficient to use only ϕ_{i1} during design in order to reduce computation time. Observe that a design that minimises the maximal value of interaction $\hat{\sigma}_{ij}$ of σ_{ij}, the settling time ϕ_{i1} and the overshoot ϕ_{i2} of σ_{ii} approximates to the ideal situation (9.5). If for the given input f, the matrix (σ_{ij}) is sufficiently close to the condition (9.5) then the desired performance (9.7) thus takes place.

The distillation control problem is often a regulator problem, in which case $f = -d$. Although no effort was made to identify d completely, observations of the steam quality and feed flow rate indicate that $d^{(1)} \neq 0$. Hence the designs given here trade-off between the functionals ϕ_{i1}, ϕ_{i2} and $\hat{\sigma}_{ij}$. It is clear from (9.3) that such a design will in general guarantee relative immunity to disturbances.

Other important factors considered during design are the constraints within which the system has to operate. For example, since linear systems theory is employed in the design, it is important that the linear range of an actuator is not exceeded. Actuator saturation is avoided by constraining the functional

$$U_i = \sup_{0 \leq t < \infty} |u_i(t)| \tag{9.8}$$

known as the maximum controller output in the ith loop. The constraint on (9.8) should ensure that the signal levels in the control loop are tolerable, that the linearisation approximation is nearly valid and that the actuator satisfies its maximum demand without undue wear. Taiwo (1979b), shows how to determine U_i for the situation when both the nonlinear model and f are known such that the linearisation is valid. However, operating experience with the distillation columns in UMIST suggests that the constraint on (9.8) should be $U_i \leq 15$.

For the system shown in Figure 9.4 let

$$\begin{aligned} \dot{x}_g(t) &= A_g x_g(t) + B_g u(t) \\ y_g(t) &= C_g x_g(t) \end{aligned} \tag{9.9}$$

be a minimal realisation of a rational approximant of $G(s)$. Also let $K(s,p)$ have the realisation

$$\begin{aligned} \dot{x}_k(t) &= A_k x_k(t) + B_k e(t) \\ u(t) &= C_k x_k(t) + D_k e(t) \end{aligned} \tag{9.10}$$

where the matrices A_k, B_k, C_k and D_k depend on the vector p. the closed-loop system therefore has the equations

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bf(t) \\ y(t) &= Cx(t) + d(t) \end{aligned} \tag{9.11}$$

where

$$A = \begin{bmatrix} A_k & -B_k C_g \\ B_g C_k & A_g - B_g D_k C_g \end{bmatrix}, \quad B = \begin{bmatrix} B_k \\ B_g D_k \end{bmatrix}, \quad C^T = \begin{bmatrix} 0 \\ C_g^T \end{bmatrix}, \quad x = \begin{bmatrix} x_k \\ x_g \end{bmatrix} \tag{9.12}$$

and superscript T denotes matrix transposition. The system is asymptotically stable if and only if all the eigenvalues λ_i(p) of A lie in the open left half plane. Thus, one of the inequalities is

$$\phi_1(p) \leq \varepsilon, \quad \varepsilon < 0 \tag{9.13}$$

where

$$\phi_i(p) = \max_i \{\mathrm{Re}[\lambda_i(p)]\} \tag{9.14}$$

and it ensures asymptotic stability of the closed-loop system. If the initial guess p^0 gives rise to a closed-loop unstable system, the MBP is used to find a stable point p by solving the inequalities (9.13). With this as the starting point, the MBP is then used to solve all the inequalities simultaneously.

Each trial of the MBP involves a test for asymptotic stability followed by the computation of ϕ_i(p) which are obtained from the step responses of

(9.11). Computation of system time responses is efficiently done by means of Zakian's (1975) I_{MN} approximants. Notice that (9.9) presupposes that the plant description, if expressed in the transfer function matrix form, is rational. A moments method of simplification (Zakian 1978, Taiwo 1995) is used to obtain a rational approximant of the plant transfer function, $G(s)$, whenever this is not rational, in order to facilitate controller design according to the above formulation.

9.2.4 Controller Design

In general, controller design is initiated by choosing a controller form and an arbitrary starting point p^0. Then, with d in (9.11) set equal to zero, system step responses are computed. The aim in controller design is to ensure that all the elements in the matrix $(\delta_{ij} - \sigma_{ij}(t))$ are close to zero for all $t > t_s$ where t_s is sufficiently short and without violating the constraint on U_i.

The functionals ϕ_{i1}, ϕ_{i2} which denote respectively the settling time and overshoot of response σ_{ii}, and $\hat{\sigma}_{ij}$, the maximal interaction, have been used here to specify system performance; the bound on θ_{i1} being largely determined by the performance of the system with non-interacting control while the bounds on θ_{i2} and $\hat{\sigma}_{ij}$ are fixed after a preliminary experimentation with the MBP.

Each element $K_{ij}(s)$ of the matrix $K(s,p)$ is restricted to be of the form of a PI controller. This choice is based on the simplicity and the widespread use of this controller form in the chemical industry.

9.2.5 Design of Controllers for the Packed Distillation Column

Since two terms in $G(s)$ contain pure time delays, the method of moments (Zakian 1978) is used to obtain a rational approximant $G^*(s)$. Minimal realisations of both $G^*(s)$ and $K(s,p)$ are then obtained in order to use the MBP according to the formulation given here.

Formulation 9.1. In this formulation we use the full PI controller (abbreviated FPI) with the transfer function matrix

$$\begin{bmatrix} p_1 + p_2 s^{-1} & p_3 + p_4 s^{-1} \\ p_5 + p_6 s^{-1} & p_7 + p_8 s^{-1} \end{bmatrix} \tag{9.15}$$

A minimal realisation of (9.15) is easily established as

$$A_k = 0_2, \qquad B_k = I_2, \qquad C_k = \begin{bmatrix} p_2 & p_4 \\ p_6 & p_8 \end{bmatrix}, \qquad D_k = \begin{bmatrix} p_1 & p_3 \\ p_5 & p_7 \end{bmatrix} \tag{9.16}$$

The following inequalities are used to specify desired performance and system constraints:

$$\begin{array}{llll} \phi_{11} \le 30, & \phi_{12} \le 0.2, & \phi_{21} \le 30, & \phi_{22} \le 0.2 \\ \hat{\sigma}_{ij} \le 0.2, & U_i \le 10,\ i = 1,2, & \phi_1 \le -0.001 & \end{array} \tag{9.17}$$

A suitable starting point is given by

$$p^0 = (-2, -1, 0.5, 0.5, 0.5, 0.5, 2, 1) \tag{9.18}$$

The performance of the system with $K(s, p^0)$ is expressed by

$$\begin{aligned} &\phi_{11} = 99, \quad \phi_{12} = 0.48, \quad \phi_{21} = 84, \quad \phi_{22} = 0.22 \\ &\hat{\sigma}_{12} = 0.25, \quad \hat{\sigma}_{21} = 0.65, \quad U_1 = 6.5, \quad U_2 = 6.3, \quad \phi_1 = -0.054 \end{aligned} \tag{9.19}$$

After 20 iterations, the MBP located

$$p = (-4, -0.968, 0.75, 0.51, 2.244, 0.266, 2.376, 1.1) \tag{9.20}$$

and the functionals of the system with $K(s, p)$ are:

$$\begin{aligned} &\phi_{11} = 45, \quad \phi_{12} = 0.2, \quad \phi_{21} = 45, \quad \phi_{22} = 0.112 \\ &\hat{\sigma}_{12} = 0.216, \quad \hat{\sigma}_{21} = 0.465, \quad U_1 = 5.8, \quad U_2 = 4.7, \quad \phi_1 = -0.054 \end{aligned} \tag{9.21}$$

We note from (9.21) that the system has not yet satisfied most of the specification. Hence starting with parameters (9.20) and seeking for ten more iterations, the MBP located

$$p = (-7.2, -1.26, 0.627, 1.97, 3.025, 0.068, 5.4, 2.179) \tag{9.22}$$

The corresponding closed-loop system functionals are now

$$\begin{aligned} &\phi_{11} = 34, \quad \phi_{12} = 0.2, \quad \phi_{21} = 29, \quad \phi_{22} = 0.114 \\ &\hat{\sigma}_{12} = 0.12, \quad \hat{\sigma}_{21} = 0.28, \quad U_1 = 7.7, \quad U_2 = 6, \quad \phi_1 = -0.054 \end{aligned} \tag{9.23}$$

Notice the large reduction in p_6 in (9.23), indicating that this element may be set equal to zero. This demonstrates one way in which the method assists the designer to determine controller structure. We therefore stipulate a controller with seven parameters in the next formation.

Formulation 9.2. p_6 is set equal to zero and a PI controller with seven parameters (abbreviated PI7), is used. A minimal realisation of this controller is:

$$A_k = 0_2, \quad B_k = I_2, \quad C_k = \begin{bmatrix} p_2 & p_4 \\ 0 & p_7 \end{bmatrix}, \quad D_k = \begin{bmatrix} p_1 & p_3 \\ p_5 & p_6 \end{bmatrix} \tag{9.24}$$

The design is continued with the parameter values just computed, *i.e.*,

$$p = (-7.2, -1.26, 0.627, 1.97, 3.025, 0.068, 5.4, 2.179) \tag{9.25}$$

The specifications are given by (9.17).

After 20 iterations the MBP located

$$p = (-7.14, -1.76, 1.72, 1.84, 4.4, 7.25, 2.73) \tag{9.26}$$

and the corresponding closed-loop system functionals are:

$$\begin{array}{lllll} \phi_{11} = 31, & \phi_{12} = 0.2, & \phi_{21} = 26, & \phi_{22} = 0.11 & \\ \hat{\sigma}_{12} = 0.167, & \hat{\sigma}_{21} = 0.24, & U_1 = 8.6, & U_2 = 7.25, & \phi_1 = -0.054 \end{array} \tag{9.27}$$

We notice from (9.27) that the only system specifications that are violated are the bounds on θ_{11} and $\hat{\sigma}_{21}$. Since these bounds are just marginally exceeded, the design is deemed satisfactory. Observe from (9.26) that the magnitudes of the proportional elements in $k_{ii}(s)$ are distinctly larger than those of $k_{ij}(s)$, $i \neq j$. This suggests that a diagonal controller may control the plant satisfactorily. Therefore we investigate this controller in the next formulation.

Formulation 9.3. Here we use the diagonal PI controller (DPI) given by the transfer function matrix

$$K(s,p) = \begin{bmatrix} p_1 + p_2 s^{-1} & 0 \\ 0 & p_3 + p_4 s^{-1} \end{bmatrix} \tag{9.28}$$

$$p^0 = (-4,\ \text{-}2,\ 5,\ 2) \tag{9.29}$$

The performance specification and constraints are given by (9.17).

After 18 iterations the MBP located

$$p = (-9.47, -2.21, 10.7, 3.44) \tag{9.30}$$

and the corresponding closed-loop system functionals are:

$$\begin{array}{lllll} \phi_{11} = 31.5, & \phi_{12} = 0.17, & \phi_{21} = 24.5, & \phi_{22} = 0.19 & \\ \hat{\sigma}_{12} = 0.36, & \hat{\sigma}_{21} = 0.24, & U_1 = 10.4, & U_2 = 10.7, & \phi_1 = -0.054 \end{array} \tag{9.31}$$

By slightly relaxing the constraints on U$_i$ the MBP was used to obtain the parameters

$$p = (-12.1, -1.95, 12.9, 4.96) \tag{9.32}$$

The corresponding functionals of the closed-loop system are now

$$\begin{array}{lllll} \phi_{11} = 36, & \phi_{12} = 0.08, & \phi_{21} = 21, & \phi_{22} = 0.21 & \\ \hat{\sigma}_{12} = 0.35, & \hat{\sigma}_{21} = 0.21, & U_1 = 12.1, & U_2 = 12.9, & \phi_1 = -0.054 \end{array} \tag{9.33}$$

$\hat{\sigma}_{ij}$ are slightly less than the values given in (9.31).

System responses with the controllers whose parameters are given in (9.26) and (9.32) and non-interacting control (Waller 1974, Taiwo 1980) are shown in Figure 9.5. As expected, the system with controller (9.24) and non-interacting control is slightly superior to that with controller (9.28) with respect to reducing interaction. However, a controller of the form (9.28) can be more cheaply implemented by means of just two conventional analogue controllers, while a computer will generally be required to implement either a controller of the form (9.24) or non-interacting control.

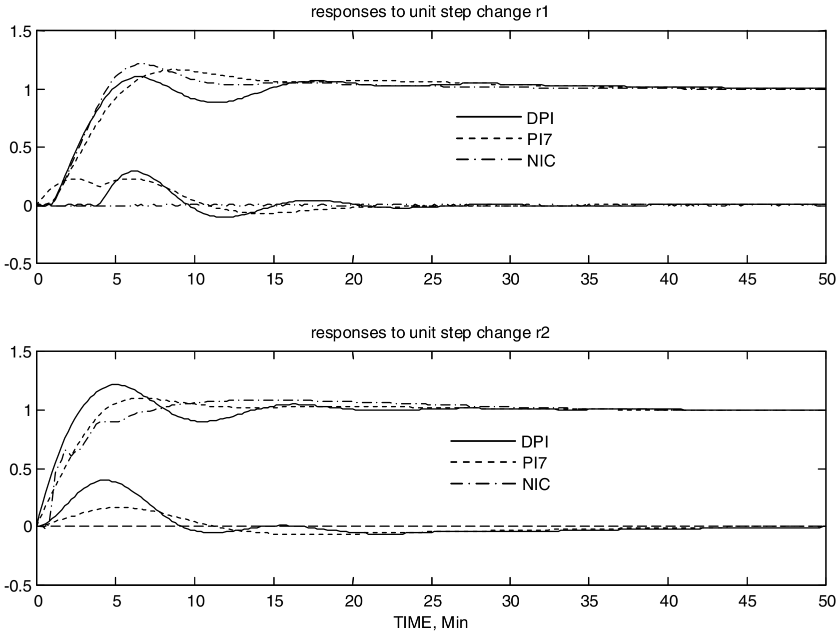

Fig. 9.5. Closed-loop responses of the packed distillation column with diagonal PI controller (DPI), PI controller with seven parameters (PI7) and non-interacting control (NIC)

9.2.6 Experimental Results

Controller (9.28) with parameters (9.32) was implemented on the plant and responses to step changes in T_{1ref} and T_{2ref} were used to compare simulation with actual plant outputs. For plant outputs during test to be distinctly more than the normal noise levels, it was necessary to introduce step changes of 1.5^oC and 2^oC in T_{1ref} and T_{2ref} respectively. Output T_1 and T_2 were then normalised before comparing with simulation (see Figure 9.6). Notice that actual plant outputs tend to 'lead" the simulated outputs. Also a great deal of irregularity is evident in the temperatures. These irregularities are more pronounced when there is a step change in T_{2ref}. Also, irregularities are confined to T_2 when there is a step change in T_{1ref}. Most of the irregularity was caused by the relatively small accuracy ($\pm 0.2°C$) and the noise problems encountered with the Kent six-point recorder. These two factors made it relatively difficult to monitor small temperature variations (*e.g.* near the steady state of T_2 and the output of T_2 due to a step change in T_{1ref}) precisely. Unfortunately the poor performance of the bottom temperature recorder also caused erratic action in the top temperature measurement. Note,

however, that the simulated responses tend to be the mean of the plant output as would be expected, since the least squares fit criterion was used to obtain the parameters of the plant model. As plant outputs were not very regular, no effort was made to implement the controller with parameters (9.26). It is worth remarking that earlier plans (Abbosh 1973) to implement an adaptive control scheme on this plant were abandoned because of the aforementioned problems.

In this section we have seen (Figure 9.5) that if the multivariable model of the column is considered during design, a diagonal controller may give satisfactory performance. However, since plant output are rather erratic, it is necessary to consider other column models in order to establish the generality of the result.

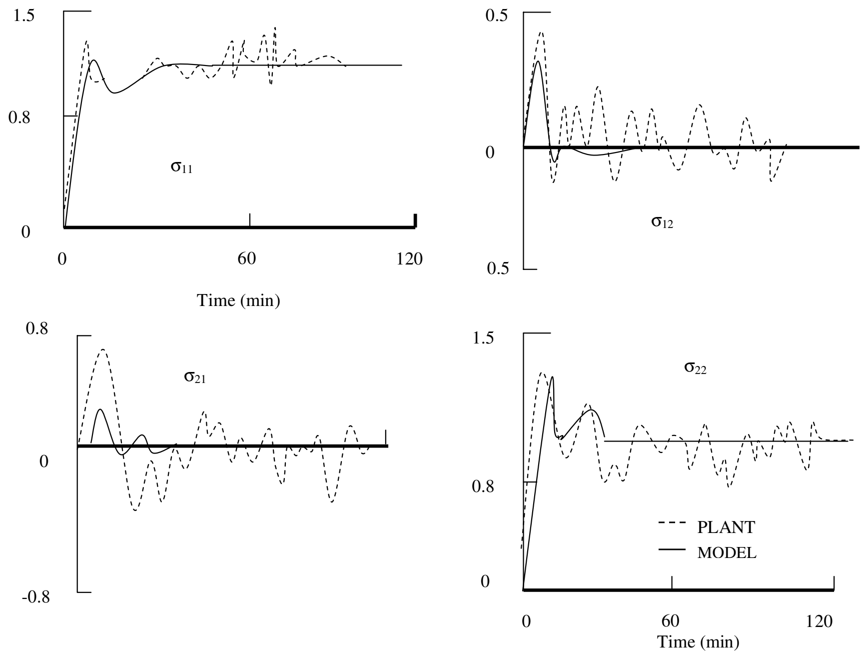

Fig. 9.6. Closed-loop responses of the packed distillation column with the diagonal PI controller whose parameters are given in (9.32)

Other distillation column models. In this section we employ the method of inequalities to design both the full PI controller and the diagonal PI controller for other distillation column models. The main motivation here is to see if the diagonal PI controller can control the column satisfactorily. Previous

investigators (Wood and Berry 1973, Luyben and Vinante 1972, Schwanke *et al* 1977) found that the closed-loop systems were very oscillatory when diagonal control was attempted. The transfer functions of these plants are given in Table 9.2.

Table 9.2. Other distillation column models

Authors	$g_{11}(s)$	$g_{12}(s)$	$g_{21}(s)$	$g_{22}(s)$
Wood and Berry (1973)	$\frac{12.8e^{-s}}{16.7s+1}$	$\frac{-18.9e^{-3s}}{21s+1}$	$\frac{6.6e^{-7s}}{10.9s+1}$	$\frac{-19.4e^{-3s}}{14.4s+1}$
Luyben and Vinante (1972)	$\frac{-2.16e^{-s}}{8s+1}$	$\frac{1.26e^{-0.3s}}{9.5s+1}$	$\frac{-2.75e^{-1.8s}}{9.5s+1}$	$\frac{4.28e^{-0.35s}}{9.2s+1}$
Schwanke, Edgar and Hougen (1977)	$\frac{-10.8(3.08s+1)}{2.13s^2+2.04s+1}$	$\frac{0.52(3.125s+1)}{1.78s^2+1.87s+1}$	$\frac{-28.14e^{-0.65s}}{1.9s^2+2.21s+1}$	$\frac{1.84}{1.87s^2+2.19s+1}$

The details of the designs are similar to those given earlier except that only diagonal PI controllers (Table 9.3) and full PI controllers (Table 9.4) are given. It is obvious in some cases which element of the full PI controllers should be set equal to zero in order to simplify the controller in a way similar to what was done above.

The responses of the system with the various controllers are shown in Figure 9.7 for the case of Wood and Berry's (1973) model. Other responses are available in Taiwo (1980) and have not been repeated here for space economy. As expected non-interacting control gives a system with the least degree of interaction.

In the case of Wood and Berry's (1973) model, the system response σ_{11} with non-interacting control is probably too oscillatory (see Figure 9.7). Better damping might result from further tuning. The controllers designed using the method of inequalities give adequately damped systems, the full PI controller resulting in less interaction. The response σ_{22} is rather sluggish when this latter controller is used. Comparison shows that the performance of the system with diagonal PI controller obtained using the method of inequalities is superior to that achieved with diagonal control in Wood and Berry (1973) and the biggest log-modulus method (Luyben and Luyben 1997) with respect to the level of interaction and adequate damping. The design in Wood and Berry (1973) (the PI controller parameters were not given) was obtained by reducing the gains of the PI controllers which had been designed for $g_{ii}(s)$ independently.

For Luyben and Vinante's model (1972), the biggest difference between the systems with non-interacting control and the full PI controller is in σ_{21}. Non-interacting control gives a system with smaller $\hat{\sigma}_{21}$. Other σ_{ij} are similar. The performances of the systems with these controllers are better than the performance of the system with a diagonal PI controller. The main fault with this latter controller is that it gives a system with larger $\hat{\sigma}_{ij}$ and σ_{11} is sluggish. Nevertheless, this diagonal PI controller gives a better system

than the diagonal PI controller designed in Luyben and Vinante (1972). The diagonal PI controller in Luyben and Vinante (1972) was obtained by using the multivariable model of the plant. Nevertheless, θ_{22} and $\hat{\sigma}_{12}$ are too large, while σ_{21} is too oscillatory.

Non-interacting control of Schwanke's model (1977), with the controller parameters given, indicates sustained small amplitude oscillations in σ_{11}. This is probably due to the presence of eight complex conjugate poles in the $g_{ij}(s)$. The system with non-interacting control gives the least amount of interaction σ_{21}, while the full PI controller gives a system which is only slightly better than that with the diagonal PI controller. However, this diagonal PI controller gives a better system, with respect to system damping and interaction, than the diagonal PI controller given in Schwanke *et al* (1977). That controller was designed using a tuning method due to Hougen (1979). The controller parameters were not given. The characteristics locus design method (MacFarlane and Belletrutti 1973) was also applied by these investigators to this plant with somewhat unsatisfactory results.

Table 9.3. Parameters of the diagonal PI controller

	p_1	p_2	p_3	p_4
Wood's model	0.1644	0.0179	-0.0581	-0.0093
Luyben's model	-1.17	-0.135	2.29	0.206
Schwanke's model	-3.54	-1.56	4.7	2.73

Table 9.4. Parameters of the full PI controller

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8
Wood's model	0.408	0.0835	-0.05557	-0.0176	0.00326	0.0334	-0.023	-0.00967
Luyben's model	-2.14	-1.04	1.12	0.326	0.5	-0.78	1.13	0.44
Schwanke's model	-1.52	-10.11	0.96	0.1	6.59	12.7	6.8	4.6

9.2.7 Conclusion

The method of inequalities has been applied to the multivariable control of four distillation columns. The method is easy to use and gives good results. As design progresses, the designer's understanding of system dynamics improves and the available trade-offs between specifications become more apparent.

The diagonal PI controllers designed by the method of inequalities give systems that are satisfactory, thus indicating that diagonal control may be acceptable with these columns. It is further shown that some previous investigators (Wood and Berry 1973, Luyben and Vinante 1972, Schwanke *et al* 1977) failed to achieve satisfactory diagonal control, either because designs

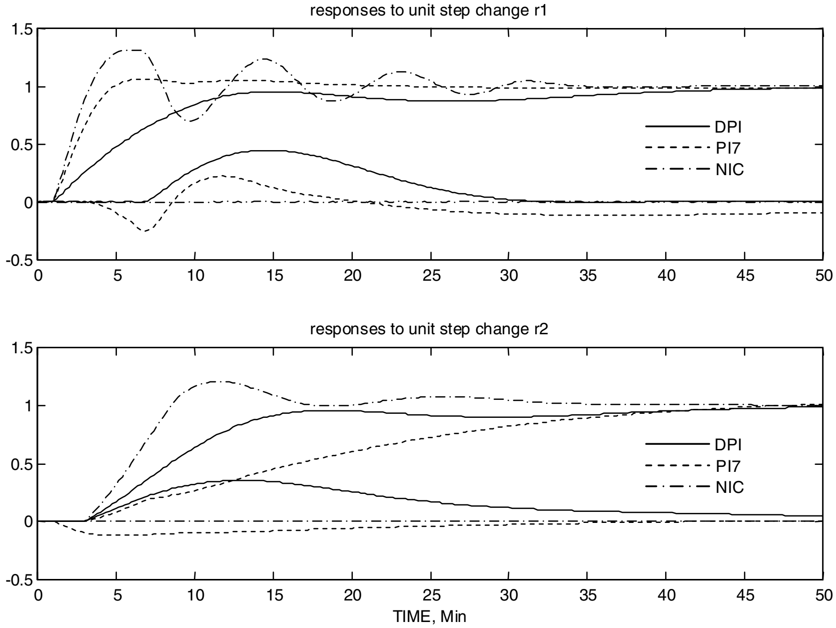

Fig. 9.7. Closed-loop responses of Wood's model with diagonal PI controller (DPI), full PI controller (FPI) and non-interacting control (NIC)

were largely based on the diagonal elements of the plant transfer function matrix, or because of the unavailability of a suitable design method for determining a satisfactory set of controller parameters. The DPI obtained here outperforms the DPI obtained using the BLT method for the Wood and Berry (1973) model. This is consistent with other similar studies (Balachandran and Chidambaram, 1996, 1997) where the DPIs designed using the method of inequalities outperform those obtained using both the BLT and the improved sequential loop closure schemes. Either the full PI controller (FPI) or non-interacting control (NIC) gives rise to somewhat better systems than the systems involving DPI. However, both the FPI and NIC are more expensive to implement and NIC is very sensitive to model inaccuracies.

9.3 Design of Multivariable Controllers for an Advanced Turbofan Engine by the Method of Inequalities

The growing complexity of aircraft engines has been a challenge to control engineers who wish to design controllers for the regulation of system out-

puts (Skira and De Hoff 1977, De Hoff and Hall 1976, Hackney *et al* 1977). It is now widely accepted that rudimentary hydromechanical control methods are not suitable for these engines. Electronic digital control, therefore is increasingly being used and many new design possibilities have received urgent attention in the industry (Sain *et al* 1978). In this work, we employ the method of inequalities to design simple multivariable controllers for the plant. The models of the engine considered here were provided by the United States National Engineering Consortium to several control groups around the world in 1977 as a form of benchmark to test the potency of the available multivariable design methods at the time. Only a few groups came up with acceptable practical solutions. The one presented here is one of the useful and simple solutions and was single-handedly obtained (Taiwo 1978, 1979). The results reveal some of the inherent difficulties associated with the control of the plant.

9.3.1 Plant Model

The 3-input, 3-output model (Sain *et al* 1978, Peczkowski 1977) of the plant are considered having orders 5 and 24, respectively. The open-loop responses of the latter model are shown in Figure 9.8.For space economy, only the design of controllers for the more difficult full order model is presented here. The reader is encouraged to consult the original work (1979b) for details of controller design for the simplified model.

The 24$^{\mathbf{th}}$ Order Model of the Plant Here the plant is considered as being made up of the turbofan engine and actuators. The engine is expressed by the linearised equations

$$\begin{aligned} \dot{x}_e(t) &= A_e x_e(t) + B_e u_e(t) \\ y_e(t) &= C_e x_e(t) + D_e u_e(t) \end{aligned} \tag{9.34}$$

where the vector y_e has components y_{e1}, y_{e2} and y_{e3} that denote, respectively, engine net thrust level, total engine air flow and engine inlet temperature, while the vector u_e has components u_{e1}, u_{e2}, and u_{e3} that denote, respectively, main burner fuel flow, nozzle jet area and inlet guide vane position. The transfer function matrix of the engine is therefore given by

$$G_e(s) = C_e(sI - A_e)^{-1} B_e + D_e \tag{9.35}$$

The matrices A_e, B_e, C_e and D_e of dimensions respectively, 16×16, 16×3, 3×16 and 3×3, are chosen such that $G_e(s)$ is the 3×3 transfer function matrix obtained by discarding the last two rows and two columns of the 5×5 transfer function matrix arising from the original 16^{th} order state space model of the engine (Sain *et al* 1978, Peczkowski 1977). The 3×3 transfer function matrix $G_a(s)$ of the actuators, which has a characteristic polynomial

of order eight, is obtained by discarding the last two rows and columns of the original 5×5 transfer function matrix model of the actuators (Sain *et al* 1978, Peckowski 1977).

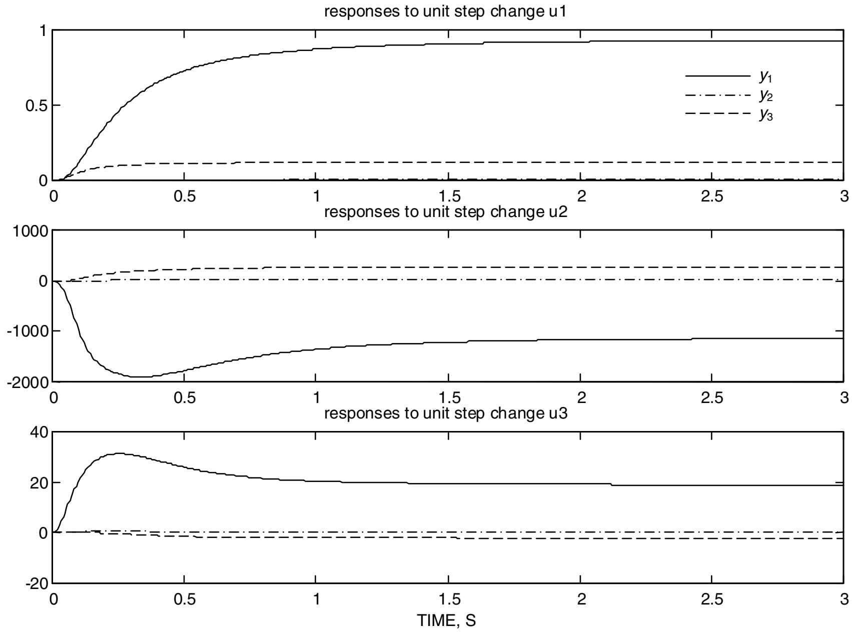

Fig. 9.8. Open-loop unit step responses of the plant

Let

$$\begin{aligned} \dot{x}_a(t) &= A_a x_a(t) + B_a u_a(t) \\ u_a(t) &= C_a x_a(t) \end{aligned} \tag{9.36}$$

be a minimal realisation of $G_a(s)$ then the plant (engine plus actuators) is given by

$$\begin{aligned} \dot{x}_c(t) &= A_c x_c(t) + B_c u_a(t) \\ y_e(t) &= C_c x_c(t) \end{aligned} \tag{9.37}$$

where

$$A_c = \begin{bmatrix} A_a & 0 \\ B_e C_a & A_e \end{bmatrix}, \quad B_c = \begin{bmatrix} B_a \\ 0 \end{bmatrix}, \quad C_c = \begin{bmatrix} D_e C_a \; C_e \end{bmatrix}, \quad x_c = \begin{bmatrix} x_a \\ x_a \end{bmatrix} \tag{9.38}$$

and u_a is a vector of controller outputs. The block diagram of the control system is shown in Figure 9.9.

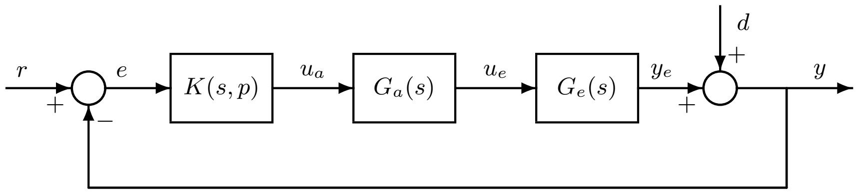

Fig. 9.9. Feedback structure for the 24^{th} order plant

It is assumed here that all the states of the engine are measurable. Measurement lags are neglected. Further details of the work reported here are available (Taiwo 1978b, 1979b).

9.3.2 Performance Specification and Constraints

When in flight, the aircraft pilot manipulates the power lever angle in order to obtain more or less thrust. The power lever angle change is converted by a master engine scheduler into reference input for the closed loop system. The nature of this reference input is not specific, but is assumed here to be a step-like function, in accordance with standard practice (Sain *et al* 1978, Peczkowski 1977, Zakian 1978). In terms of our linear models this control problem can be summarised thus : with a step change in power lever angle we want to move the engine to the appropriate operating point quickly with the plant variables responding smoothly (Sain *et al* 1978, Peczkowski 1977, Hackney *et al* 1977).

The specifications are therefore that the closed-loop system should be approximately decoupled and the net thrust level, y_{e1}, and the inlet temperature, y_{e3}, should respond quickly and with little overshoot. The maximal value of the modulus of the ith input to the engine is

$$U_{ei}(t) = \sup_{0 \leq t \leq \infty} |u_{ei}(t)| \tag{9.39}$$

This is constrained to be less than 1 ft and 6° for $i = 2$ and 3, respectively, for unit step changes, applied individually, in the reference inputs. Also, the bounds on the maximal rate of change

$$V_{ei}(t) = \sup_{0 \leq t \leq \infty} \left| \frac{du_{ei}(t)}{dt} \right| \tag{9.40}$$

of these inputs are respectively 15800(lb/hr)/s, 3.6 ft/s and 48°/s for $i = 1, 2, 3$ for unit step changes, applied individually, in the reference inputs.

9.3.3 The Design of Controllers

Formulation 9.4. The controllers designed for the reduced fifth order model give an unstable closed-loop system when implemented on the 24th order model (Taiwo 1979b). Computations using the MBP showed that in order to obtain an acceptable design the multivariable PI controller

$$K(s,p) = \begin{bmatrix} p_1 + p_2 s^{-1} & p_3 + p_4 s^{-1} & p_5 + p_6 s^{-1} \\ 0 & p_7 s^{-1} & 0 \\ p_8 & 0 & p_9 s^{-1} \end{bmatrix} \tag{9.41}$$

which has a minimal realisation

$$A_k = 0_3, \quad B_k = I_3, \quad C_k = \begin{bmatrix} P_2 & P_4 & P_6 \\ 0 & P_7 & 0 \\ 0 & 0 & p_y \end{bmatrix}, \quad D_k = \begin{bmatrix} P_1 & P_3 & P_5 \\ 0 & 0 & 0 \\ P_8 & 0 & 0 \end{bmatrix} \tag{9.42}$$

may be used. Both theoretical considerations and observed system responses indicate that the closed-loop system with controller (9.42) gives correct steady state responses. Therefore, in the subsequent design, only inequalities relating to the settling time, overshoot, U_{ei}, V_{ei} and the interaction functionals were considered.

In the course of the design it became clear that a conflict exists between the requirements of small interaction and adequate damping. The parameters that give a reasonable compromise between these conflicting requirement were found to be

$$p = (-7.74, 80.39, 7.98, 81.6, 47.69, 293.71, 0.03, 0.32, -7.77) \tag{9.43}$$

and the functionals of the closed-loop system with $K(s,p)$ are

$$\begin{array}{llll} \text{Settling time} & 1.28 & 2.7 & 1.5 \\ \text{Overshoot} & 0.2 & 0 & 0.35 \\ & \hat{\sigma}_{21} = 0.01 & \hat{\sigma}_{12} = 0.13 & \hat{\sigma}_{13} = 0.018 \\ & \hat{\sigma}_{31} = 0.013 & \hat{\sigma}_{32} = 0.13 & \hat{\sigma}_{23} = 0.039 \\ & U_{e1} = 12.6 & U_{e2} = 0.023 & U_{e3} = 2.3 \\ & V_{e1} = 60 & V_{e2} = 0.03 & V_{e3} = 2.8 \end{array} \tag{9.44}$$

where in (9.44) and the rest of the chapter, the numerical values in the first column are functionals of σ_{11}, while those in the second column are functionals of σ_{22}, and those in the third column are functionals of σ_{33}.

The three pairs of complex conjugate poles $(-6.04 \pm j71.37, -5.9 \pm j40.45, -0.86 \pm j6.07)$ cause this system to exhibit some transient oscillation. Also, the overshoots are rather large. This leads to the next formulation.

Formulation 9.5. The controller considered in this formulation is

$$K(s,p) = \begin{bmatrix} (p_1 + p_2 s^{-1}) \left(\frac{s+p_{10}}{s+p_{11}}\right) & p_3 + p_4 s^{-1} & p_5 + p_6 s^{-1} \\ 0 & p_7 s^{-1} & 0 \\ p_8 \left(\frac{s+p_{10}}{s+p_{11}}\right) & 0 & p_9 s^{-1} \end{bmatrix} \tag{9.45}$$

This structure gives a big improvement in the transients of σ_{11}. An acceptable set of controller parameters was found to be

$$p = (6.58, 402.4, 7.65, 77.78, 66.7, 477, 0.0155, 0.37, -7.4, 2.16, 57.1) \tag{9.46}$$

The closed-loop system with $K(s,p)$ has the following functionals:

$$\begin{array}{llll} \text{Settling time} & 1.28 & 2.7 & 1.5 \\ \text{Overshoot} & 0.2 & 0 & 0.35 \\ & \hat{\sigma}_{21} = 0.01 & \hat{\sigma}_{12} = 0.13 & \hat{\sigma}_{13} = 0.018 \\ & \hat{\sigma}_{31} = 0.013 & \hat{\sigma}_{32} = 0.13 & \hat{\sigma}_{23} = 0.039 \\ & U_{e1} = 12.6 & U_{e2} = 0.023 & U_{e3} = 2.3 \\ & V_{e1} = 60 & V_{e2} = 0.03 & V_{e3} = 2.8 \end{array} \tag{9.47}$$

This controller succeeds in almost eliminating the overshoot in σ_{33}. However, the overshoot in σ_{11} is rather large. There is also some oscillation in σ_{33}, due mainly to the eigenvalue $-2.97 \pm j46.84$ of the matrix A. As in Formulation 9.4, this design represents a compromise between small iteration and adequate damping. In order to improve σ_{33} also, we now consider another formulation.

Formulation 9.6. The controller considered here is

$$K(s,p) = \begin{bmatrix} (p_1 + p_2 s^{-1}) \left(\frac{s+p_{10}}{s+p_{11}}\right) & p_3 + p_4 s^{-1} & (p_5 + p_6 s^{-1}) \left(\frac{s+p_{12}}{s+p_{13}}\right) \\ 0 & p_7 s^{-1} & 0 \\ p_8 \left(\frac{s+p_{10}}{s+p_{11}}\right) & 0 & p_9 s^{-1} \left(\frac{s+p_{12}}{s+p_{13}}\right) \end{bmatrix} \tag{9.48}$$

The closed-loop system with

$$p = (7.18, 75.74, 62.96, 48.4, 276.7, 0.029, 0.45, -8.55, 8.64, 15.24, 16.45, 22.81) \tag{9.49}$$

has the following functionals:

$$\begin{array}{llll} \text{Settling time} & 1.6 & 2.0 & 1.75 \\ \text{Overshoot} & 0.54 & 0 & 0.25 \\ & \hat{\sigma}_{21} = 0.03 & \hat{\sigma}_{12} = 0.44 & \hat{\sigma}_{13} = 0.27 \\ & \hat{\sigma}_{31} = 0.04 & \hat{\sigma}_{32} = 0.41 & \hat{\sigma}_{23} = 0.07 \\ & U_{e1} = 12.6 & U_{e2} = 0.023 & U_{e3} = 2.3 \\ & V_{e1} = 49.5 & V_{e2} = 0.03 & V_{e3} = 2.8 \end{array} \tag{9.50}$$

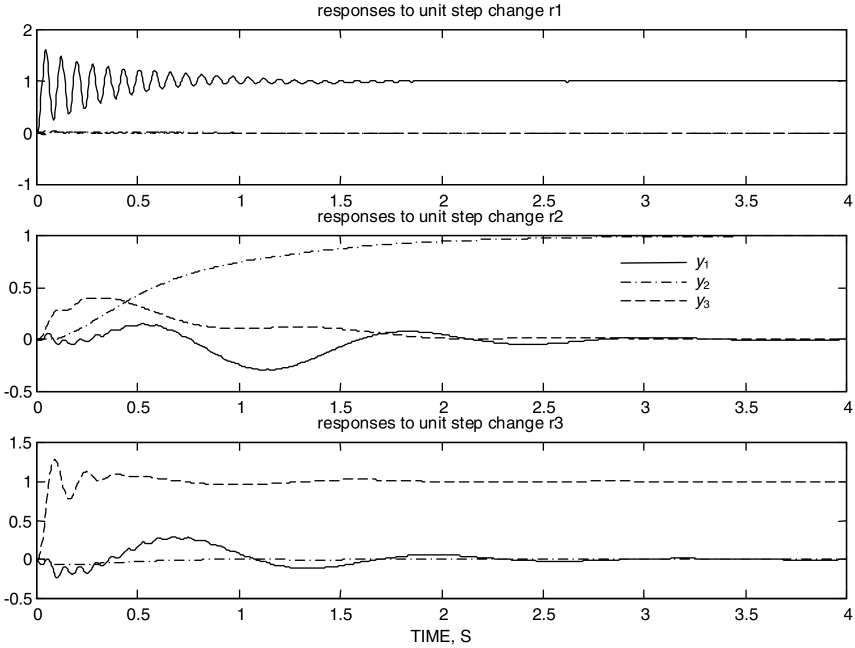

Fig. 9.10. Closed-loop responses of the system with controller parameters (9.49)

The system responses shown in Figure 9.10 clearly demonstrate that the system is lightly damped, though interaction is small. The oscillations are caused by the presence of the eigenvalues $(2.42 \pm j81.73, -8.9 \pm j41.35, -1.2 \pm j5.09$ in the matrix A.

However, with

$$p = (-7.52, 115.3, 9.38, 59.82, 297.7, 0.028, 0.3, -6.52, 2.28, 21.53, 14.52, 40.28) \tag{9.51}$$

the closed-loop system with $K(s,p)$ is reasonably damped. Nevertheless, the interaction level has now increased. The functionals of the closed-loop system are:

$$\begin{array}{llll} \text{Settling time} & 1.28 & 2.7 & 1.5 \\ \text{Overshoot} & 0.2 & 0 & 0.35 \\ & \hat{\sigma}_{21} = 0.02 & \hat{\sigma}_{12} = 0.78 & \hat{\sigma}_{13} = 0.4 \\ & \hat{\sigma}_{31} = 0.04 & \hat{\sigma}_{32} = 0.93 & \hat{\sigma}_{23} = 0.05 \\ & U_{e1} = 12.6 & U_{e2} = 0.023 & U_{e3} = 2.3 \\ & V_{e1} = 69 & V_{e2} = 0.03 & V_{e3} = 2.7 \end{array} \tag{9.52}$$

The closed-loop system responses are shown in Figure 9.11. The oscillations noticed in the responses are due mainly to the eigenvalues ($-13.8 \pm j73.7, -13.68 \pm j45.28, -1.56 \pm j5.81$) in the matrix A. Comparisons with the open-loop system clearly show the merits of the various designs.

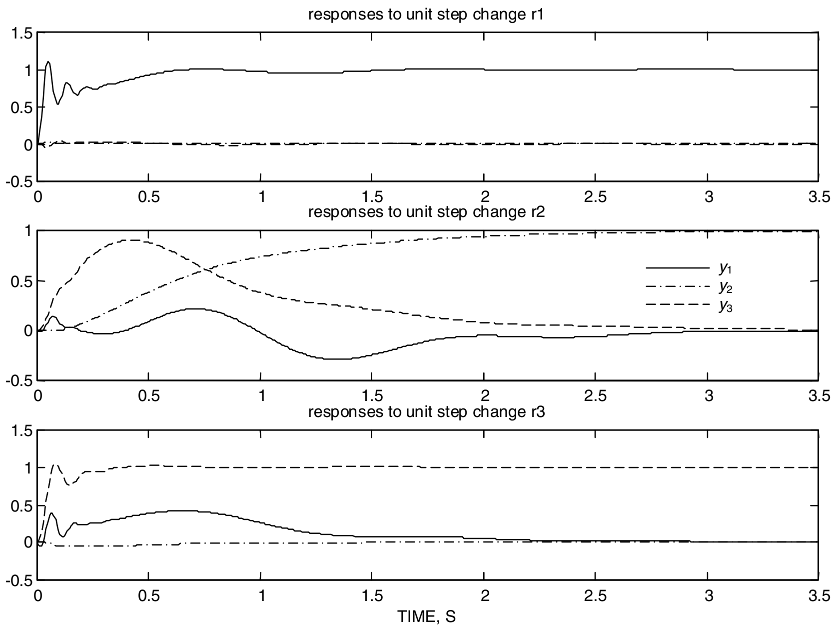

Fig. 9.11. Closed-loop responses of the system with controller parameters (9.51)

9.3.4 Conclusions

The method of inequalities is used to design controllers for the regulation of net thrust level, total air flow and inlet temperature of an F100 turbofan engine. Two models of the plant are considered, having orders 24 and 5, respectively, and simple controllers are designed for each model. As expected, the 24$^{\text{th}}$ order model is more difficult to control than the simplified 5$^{\text{th}}$ order model. In fact, it is found that the simplified model can give misleading results. Several useful features of the method are illustrated by this application. Most of the difficulty encountered in the design was a result of the sluggish response of the total engine airflow and the interaction which exists when there is a step change in the reference value of the total engine air flow. There is a conflict between little interaction and adequate damping.

9.4 Improvement of Turbo-alternator Response by the Method of Inequalities

The design of a control system for a turbo-alternator connected to an infinite bus through a transmission line has been the subject of several studies aimed at finding simple controllers which meet the usual design specifications. Ahson and Nicholson (1976) applied the Inverse Nyquist Array method (Rosenbrock 1974) to obtain a feedback controller for regulating the terminal voltage and load angle of a turbo-alternator. They employ a ninth-order model of the turbo-alternator (Davison and Rau 1971, Taiwo 1978) which takes the form

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{9.53}$$

and design a system having a closed-loop transfer function matrix $H(s)$ given by

$$H(s) = [I + L(s)G(s)K(s)F(s)]^{-1}L(s)G(s)K(s) \tag{9.54}$$

where $G(s)$ is the transfer function matrix of the plant, *i.e.*,

$$G(s) = C(sI - A)^{-1}B \tag{9.55}$$

and $K(s)$, $L(s)$, $F(s)$ are the various matrices that make up the controller and called respectively the pre, post and feedback compensators. The results they obtain are expressed by:

$$K(s) = \begin{bmatrix} \frac{10s^2+1001s+100}{s^2+100s} & 0 \\ \frac{0.7s^2+70s+7}{s^2+100s} & \frac{30s+100}{s+100} \end{bmatrix} \tag{9.56}$$

$$L(s) = \begin{bmatrix} 1 & 0 \\ 0 & 0.1 \end{bmatrix}, \qquad F(s) = \begin{bmatrix} 100 & 0 \\ 0 & 0.2 \end{bmatrix} \tag{9.57}$$

Ahson and Nicholson comment upon the advantages of the Inverse Nyquist Array method, and point out that the controller they obtain is simpler than those previously obtained by means of optimal control techniques and can be readily implemented by available hardware.

The present study begins with the observation that the result of Ahson and Nicholson does not quite meet the main performance objectives with respect to steady–state and transient behaviour and stability margin. Furthermore, it is shown that by using the method of inequalities very simple controllers are found which meet the performance specifications. It is well-known that the turbo-alternator problem gives rise to severe difficulties. The results of this section throw some light on the nature of these difficulties.

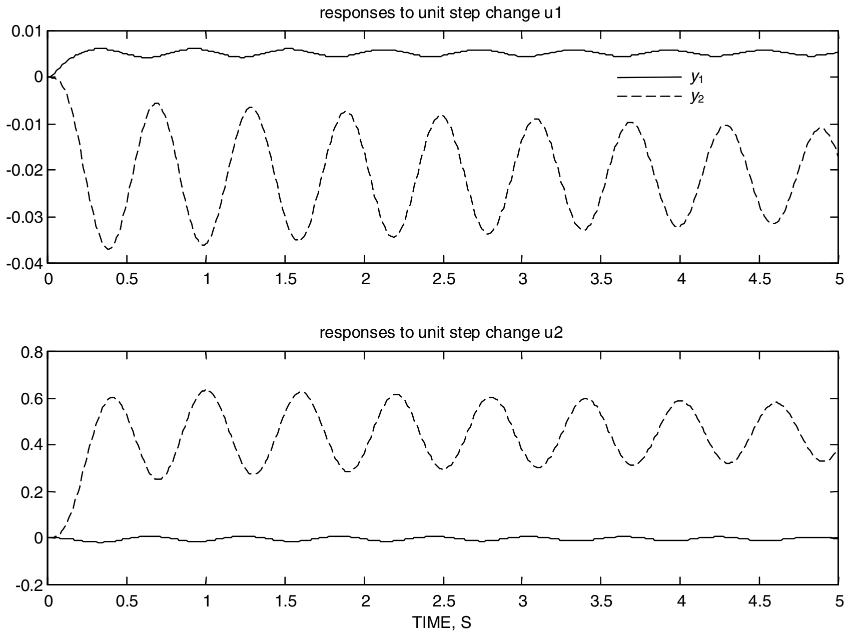

Fig. 9.12. Open-loop responses of the turbo-alternator

9.4.1 Performance

The performance required of a multivariable regulator can be summarised as follows. To a step change applied to any one reference variable, outputs respond by tending smoothly and rapidly to the new values of their respective reference variables. The above statement implies that the closed-loop transfer function matrix $H(s)$, has to satisfy a number of conditions. For correct steady-state performance, $H(s)$ has to satisfy the condition

$$H(0) = I \tag{9.58}$$

The design of Ahson and Nicholson gives

$$H(0) = \begin{bmatrix} 0.01 & 0.0 \\ 0.0021 & 0.0448 \end{bmatrix} \tag{9.59}$$

The transient output changes caused by a step input should not display lightly damped oscillations; or, equivalently, the eigenvalues of the A matrix of the minimal realisation of $H(s)$ should not be located too near the line of imaginaries. In fact the design of Ahson and Nicholson does not satisfy these requirements although, as they demonstrate, their system behaves considerably better in these respects than the uncompensated system. To verify these remarks, the reader should refer to the details in their paper.

9.4.2 Design of a Turbo-alternator Controller

Appropriate inequalities were chosen in accordance with the performance requirements prescribed by Ahson and Nicholson (1976). It may be recalled here that the abscissa of stability of a multivariable system is the largest of the real parts of the eigenvalues of the A matrix of the minimal realisation of the closed loop transfer function matrix $H(s)$.

The controller was chosen to be of the form such that $L(s) = F(s) = I$ and $K(s,p)$ is the simplest matrix having a vector p of the least dimension. The simplest controller which satisfies all the requirements is found to be

$$K(s,p) = \frac{1}{s}\begin{bmatrix} 78.6 & 67.1 \\ 3.8 & 5.1 \end{bmatrix} \tag{9.60}$$

With this controller the transient responses of the system caused by unit step changes in the reference variables are as shown in Figure 9.13. It is clear from this that the design meets the requirements of transient and steady state performance. The abscissa of stability is -0.2.

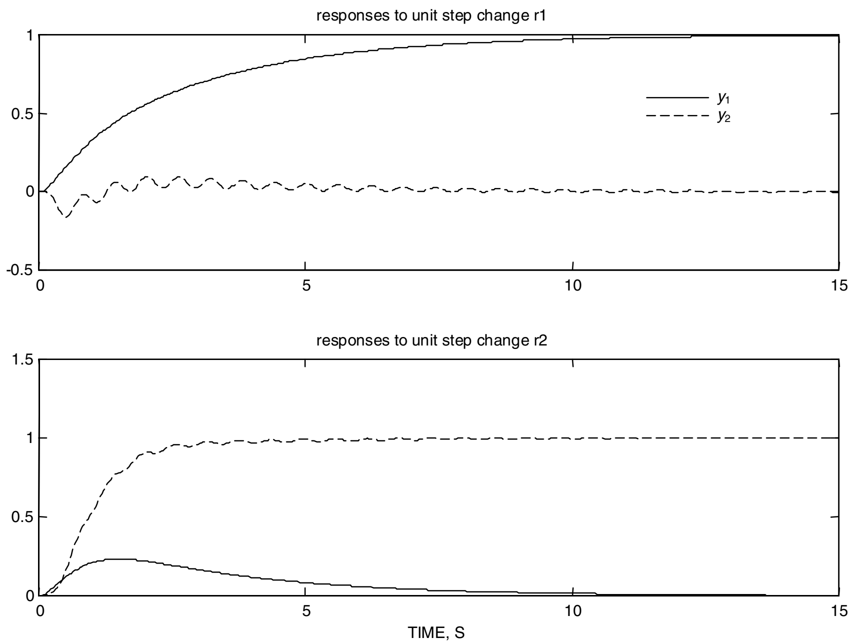

Fig. 9.13. Closed-loop responses of the turbo-alternator with controller (9.60)

For this model of the turbo-alternator, it is found that a conflict exists between the stability margin and interaction. It is also found that for ac-

ceptable interaction, increasingly more complicated controllers are required in order to improve stability margin. Controllers of the form

$$K(s,p) = \begin{bmatrix} k_{11}(s) & 0 \\ k_{21}(s) & k_{22}(s) \end{bmatrix} \tag{9.61}$$

was found to give rise to a system with very low interaction. However, this structure was found to produce a system with a relatively small stability margin for small interactions. Even with each $k_{ij}(s)$ in the form of a proportional-plus-integral controller, it was found that unacceptable interaction results for abscissas of stability as low as -0.1. The simplest controller of the form of (9.61) which satisfies the requirements, and with an abscissa of stability of -0.07 is

$$K(s) = \frac{1}{s}\begin{bmatrix} 36 & 0 \\ 1.7 & 0.47 \end{bmatrix} \tag{9.62}$$

Note that the stability margin in this case is identical with that of the design by Ahson and Nicholson (1976). The controller

$$K(s,p) = \frac{1}{s}\begin{bmatrix} 143.7(1+0.3s) & 31.7(1+0.47s) \\ 6.1(1+0.56s) & 5.5 \end{bmatrix} \tag{9.63}$$

gives rise to a system with the better abscissa of stability of -1.25. These designs clearly demonstrate the trade-offs that can take place between controller complexity (*i.e.* cost) and system performance.

9.4.3 Conclusion

This section summarises, very briefly, the work of Taiwo (1978). The method of inequalities has been used to design very simple controllers for the plant. The main difficulty found in designing controllers for the plant lies in the conflict which exists between stability margin and interaction. For an acceptable level of interaction, it is found that a greater stability margin is obtained only at the expense of increasing the complexity of the controller.

Notation

A	closed-loop system stability matrix
d	disturbance variable vector
f	system input $(r - d)$
$f^{(1)}(t)$	$(df(t))/(dt)$
G	transfer function matrix of the plant
I_ℓ	identity $\ell \times \ell$ matrix
J	$\{1, 2, \ldots, \ell\}$
K	transfer function matrix of the controller

ℓ	general designation for the number of inputs (or outputs) of a square system
DPI	diagonal proportional plus integral controller (Equation (9.28))
FPI	full PI controller (Equation (9.15))
PI7	proportional plus integral controller with seven parameters (Equation (9.24))
MBP	Moving Boundaries Process
NIC	non-interacting control
R	change in reflux flowrate (Kg/hr)
r	reference variable vector
S	change in steam flowrate (Kg/hr)
s	Laplace transform variable
T_1, T_{1ref}	upper temperature (or composition), upper temperature (or composition) set point
T_2, T_{2ref}	lower temperature (or composition), lower temperature (or composition) set point

U_i defined in (9.8)

Greek symbols

δ_{ij}	Kronecker delta, *i.e.*, $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ if $i \neq j$
σ_{ij}	ith component (y_i or T_i) of the closed-loop system output vector due to a unit step change in r_j or T_{jref}
$\hat{\sigma}_{ij}$	$\max\limits_{0 \leq t < \infty} \frac{\lvert\sigma_{ij}(t)\rvert}{\sigma_{jj}(\infty)}$, $i \neq j$, $i, j \in J$
ϕ_{i1}	settling time of σ_{ii}, defined as the least value of t_1 such that $\lvert\sigma_{ii}(\infty) - \sigma_{ii}(t)\rvert \leq 0.02 \lvert\sigma_{ii}(\infty)\rvert, \forall\, t > t_1$
ϕ_{i2}	overshoot of σ_{ij}, given by $(\hat{\sigma}_{ii} - \lvert\sigma_{ii}(\infty)\rvert)/\lvert\sigma_{ii}(\infty)\rvert$ if $\hat{\sigma}_{ii} > \lvert\sigma_{ii}(\infty)\rvert$ and zero if $\hat{\sigma}_{ii} \leq \lvert\sigma_{ii}(\infty)\rvert$ where $\hat{\sigma}_{ii} = \max\limits_{0 \leq t < \infty} \lvert\sigma_{ii}(t)\rvert$
λ_i	eigenvalue of A
ϕ_1	defined in (9.13)

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